

Question Paper Quantitative Methods – I (131): January 2005
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| <ul style="list-style-type: none">• • Answer all questions.• • Marks are indicated against each question. |
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1. The average deviation of all items in the data from zero is equal to the
- (a) Arithmetic mean (b) Median
(c) Mode
(d) Standard deviation (e) Variance.

(1 mark)

2. If every item in a data set is increased by a constant C , then the arithmetic mean of the resulting data set will be equal to
- (a) The mean of the original data set (b) $C + \text{Mean of the original data set}$
(c) $C - \text{Mean of the original data set}$ (d) $\text{Mean of the original data set} \div C$
(e) $\text{Mean of the original data set} \times C$.

(1 mark)

3. Which of the following is a relative measure of dispersion?
- (a) Variance (b) Standard deviation
(c) Range
(d) Coefficient of variation (e) None of the above.

(1 mark)

4. If every item in a data set is decreased by the same quantity then the standard deviation of the resulting data set

- (a) Remains the same
- (b) Increases by the same quantity by which every data item is increased
- (c) Decreases by the same quantity by which every data item is increased
- (d) Increases by the square root of the same quantity by which every data item is increased
- (e) Decreases by the square root of the same quantity by which every data item is increased.

(1 mark)

5. The appropriate mean for a set of ratios using the denominators of the ratios as weights is

- (a) Geometric mean
- (b) Simple harmonic mean
- (c) Weighted arithmetic mean
- (d) Median
- (e) Mode.

(1 mark)

6. Which of the following measures **cannot** be combined mathematically?

- (a) Standard deviation
- (b) Arithmetic mean
- (c) Geometric mean
- (d) Harmonic mean
- (e) Median.

(1 mark)

7. The standard deviation of a data set

- (a) Is expressed in the same unit as the observations in the data set
- (b) Is expressed in the square of the unit of the observations in the data set
- (c) Is expressed in the square root of the unit of the observations in the data set
- (d) Is expressed in a different unit from the unit in which the observations in the data set are expressed

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(e) Is always expressed as a percentage of the mean of the data set.

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8.

The coefficient of variation cannot be meaningfully used to compare the variability of two or more sets of data, when

- (a) The standard deviation is zero for one or more sets of data
- (b) The standard deviation is 1 for one or more sets of data
- (c) The mean is zero for one or more sets of data
- (d) The mean is 1 for one or more sets of data
- (e) The mean and standard deviation are equal for one or more sets of data.

(1 mark)

9.

Which of the following is **true**, if there is no dispersion in a data set?

- (a) All the mathematical and positional averages are equal
- (b) All the mathematical averages are equal but the positional averages are not equal
- (c) All the positional averages are equal but the mathematical averages are not equal
- (d) All the mathematical averages are equal to zero
- (e) All the positional averages are equal to zero.

(1 mark)

10.

Which of the following statements is **true**?

- (a) The tallest rectangle in a histogram represents the modal class of the distribution
- (b) In a symmetrical distribution the mean, median and mode are unequal
- (c) The medians of two sets of data can be combined

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- (d) The median can not be determined graphically
- (e) The mode is always uniquely defined.

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11. Which of the following statements is **true** with regard to a linear programming problem?

- (a) The objective function is a statement of a constraint
- (b) The coefficients of the decision variables in the constraints represent the per unit contribution of the decision variables to the value of the objective function
- (c) The decision variables can not assume zero values
- (d) The constraint inequations specify the consumption of resources and the amount of resources available
- (e) The coefficients of the decision variables in the objective function represent the per unit consumption of resources by the decision variables.

(1 mark)

12. Which of the following is **not** an assumption underlying linear programming?

- (a) The objective to be accomplished can be expressed as a linear function of the decision variables
- (b) The constraints on the use of resources can be expressed as linear equations or inequations
- (c) The amount of resources consumed by each unit of the decision variables is uncertain
- (d) The decision variables can take non-negative values only
- (e) The total consumption of a resource is the sum of the resources consumed by each decision variable.

(1 mark)

13. Which of the following is **false** with regard to the

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graphical method of solving linear programming problems?

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- (a) The decision variables are represented by the horizontal and vertical axes
- (b) The graphical method can be applied when there are two decision variables
- (c) The optimal solution, if it exists, occurs at one of the corner points of the feasible region
- (d) The optimal solution to the problem always occurs at a point outside the feasible region
- (e) The feasible region is determined by the structural constraints and non-negativity constraints.

(1 mark)

14. Which of the following is **true** with regard to the simplex method of solving a linear programming problem (LPP) on profit maximization?

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- (a) At the optimal solution the slack variables are always equal to zero
- (b) The constraints which contain $\leq$ are converted into equations by adding slack variables
- (c) There can be only one feasible solution to a LPP
- (d) The slack variables make positive contributions to profit
- (e) The slack variables can only assume the values 0 or 1.

(1 mark)

15. Which of the following conditions indicates the existence of multiple optimal solutions when a linear programming problem is solved by the graphical method?

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- (a) One of the constraints is parallel to the horizontal axis

- (b) The objective function is parallel to the vertical axis
- (c) The objective function is parallel to one of the edges of the feasible region which is in the direction of optimal movement of the objective function
- (d) All the decision variables assume negative values
- (e) One of the decision variables assumes negative values.

(1 mark)

16. Which of the following is **true** with regard to the classical approach to probability?

- (a) It assumes that the outcomes are not equally likely
- (b) The probability of an event is determined after performing the experiment large number of times
- (c) The probability of an event is determined before performing the experiment
- (d) It assumes that all possible outcomes of the experiment are not known
- (e) The classical approach cannot be used to find out the probability of mutually exclusive events.

(1 mark)

17. If events **A** and **B** are mutually exclusive then which of the following is **true**?

- (a) $P(A \text{ and } B) = 0$
- (b) $P(A) = 0$
- (c) $P(B) = 0$
- (d) $P(A \text{ or } B) = 0$
- (e) $P(A \text{ or } B) = 1$.

(1 mark)

18. The probability of occurrence of an event is expressed as a number which lies between

- (a) 0 and 1
- (b) 1 and 2
- (c) -1 and 0
- (d) -2 and -1
- (e) 1 and infinity.

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19. If a function, $f(x)$, has a relative maxima at a point $x = c$, then, which of the following is **true**?

- (a) The first order derivative of $f(x)$ at $x = c$ is positive
- (b) The first order derivative of $f(x)$ at $x = c$ is negative
- (c) The second order derivative of $f(x)$ at $x = c$ is zero
- (d) The second order derivative of $f(x)$ at $x = c$ is positive
- (e) The second order derivative of $f(x)$ at $x = c$ is negative.

(1 mark)

20. Which of the following is **false** with regard to the derivative of a function?

- (a) It indicates the rate of change of the function at a given point
- (b) The slope of the tangent to a function at a point is equal to the derivative of the function at the point
- (c) The derivative may be a function of the independent variable
- (d) The derivative of a linear function changes with the value of the independent variable
- (e) If the derivative of a function at a point is negative then it indicates that the function is decreasing at that point.

(1 mark)

21. For a function $f(x)$ the first derivative is positive and the second derivative is negative at $x = a$. This means that

- (a) $f(x)$ is minimum at $x = a$
- (b) $f(x)$ is maximum at $x = a$
- (c) $f(x)$ is increasing at an increasing rate at $x = a$
- (d) $f(x)$ is increasing at a decreasing rate at $x = a$

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(e) $f(x)$ is decreasing at an increasing rate at $x = a$.

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22.

A function $f(x)$ is said to be monotonically increasing if

(a) The first derivative of $f(x)$ is a constant for all values of x

(b) The first derivative of $f(x)$ is negative for all values of x

(c) The first derivative of $f(x)$ is positive for all values of x

(d) The first derivative of $f(x)$ is zero for all values of x

(e) The first derivative of $f(x)$ is equal to 1 for all values of x .

(1 mark)

23.

For any function $f(x)$ the limit of $f(x)$ as x approaches a value a is

(a) Always equal to a

(b) Always a value greater than a

(c) Always a value less than a

(d) Always less than $f(a)$

(e) A value L , such that $f(x)$ approaches L as x approaches a .

(1 mark)

24.

A function is a rule or correspondence which always associates to

(a) Each number x in a set A a unique number $f(x)$ in a set B

(b) All the numbers in a set A the same number in a set B

(c) All the numbers in a set A a fixed positive number

(d) All the numbers in a set A a fixed negative number

(e) Each number x in a set A , two numbers in a set B .

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25. Which of the following is **not** an unbounded interval?
- (a) The set of all real numbers greater than a specified real number
 - (b) The set of all real numbers less than a specified real number
 - (c) The set of all real numbers greater than or equal to a specified real number
 - (d) The set of all real numbers less than or equal to a specified real number
 - (e) The set of all real numbers greater than one real number and less than another real number.

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26. The reciprocals of the terms in a harmonic progression are
- (a) In geometric progression
 - (b) In harmonic progression
 - (c) In arithmetic progression
 - (d) Always in decreasing order
 - (e) Always in increasing order.

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27. If the first term in a geometric progression is greater than 1 and the common ratio is less than 1, then the consecutive terms will be
- (a) The consecutive terms will be in increasing order
 - (b) The consecutive terms will be in decreasing order
 - (c) The consecutive terms will be same
 - (d) All the consecutive terms will be less than 1
 - (e) All the consecutive terms will be greater than 1.

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28. If all the terms of an arithmetic progression are

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multiplied by a constant quantity the resulting terms will always form

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- (a) A geometric progression
- (b) A harmonic progression
- (c) An arithmetic progression
- (d) Either a geometric progression or a harmonic progression
- (e) Either a geometric progression or an arithmetic progression.

(1 mark)

29.

The set of whole numbers includes the set of

- (a) Natural numbers
- (b) Rational numbers
- (c) Negative numbers
- (d) Complex numbers
- (e) Irrational numbers.

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30.

According to the inverse property of addition

- (a) For every real number there exists another real number such that the sum of the two real numbers is equal to 1
- (b) For every real number there exists another real number such that the sum of the two real numbers is equal to 0
- (c) The addition of zero to any real number is equal to that real number
- (d) For every real number there exists another real number such that the product of the two numbers is equal to 1
- (e) The product of any real number with 1 is equal to that real number.

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31.

In a positively skewed distribution

- (a) Majority of the observations are concentrated towards the higher end of the scale
- (b) Majority of the observations are concentrated towards the lower end of the scale
- (c) Majority of the observations are concentrated at the center of the distribution
- (d) The observations have the same frequency
- (e) The distribution of the data is symmetrical.

(1 mark)

32.

Which of the following measures is based only on two observations in a data set?

- (a) Arithmetic mean
- (b) Harmonic mean
- (c) Range
- (d) Mean absolute deviation
- (e) Standard deviation.

(1 mark)

33.

Which of the following measures is **not** affected by the presence of extreme values in a data set?

- (a) Range
- (b) Arithmetic mean
- (c) Standard deviation
- (d) Variance
- (e) Median.

(1 mark)

34.

If every observation in a data set is increased by a constant quantity then the coefficient of variation of the resulting set of values will be

- (a) Less than the coefficient of variation of the original data set
- (b) Greater than the coefficient of variation of the original data set
- (c) Equal to the coefficient of variation of the original

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- (d) Equal to the coefficient of variation of the original data set plus the square root of the constant quantity
- (e) Equal to the coefficient of variation of the original data set multiplied by the square root of the constant quantity.

(1 mark)

35. If two events **A** and **B** are independent then, the conditional probability of event **A** given that event **B** has occurred, is equal to

- (a) Joint probability of events **A** and **B**
- (b) Conditional probability of event **B** given event **A**
- (c) Marginal probability of event **B**
- (d) Marginal probability of event **A**
- (e) Zero.

(1 mark)

36. The following details are available with regard to a sample of ten observations:

Arithmetic mean of the first four observations = 16

Arithmetic mean of the next six observations = 11

$$\sum_{i=1}^{10} x_i^2 = 1726$$

What is the standard deviation of the sample?

- (a) 2
- (b) 3
- (c) 4
- (d) 5
- (e) 6.

(1 mark)

37. The following details are available with regard to two groups of data, **A** and **B**:

Group	Number of observations	Mean	Standard deviation
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A	15	20	4
B	25	16	2

What is the combined standard deviation for both the groups?

- (a) 2.50 (b) 3.50 (c) 6.25 (d) 7.00
(e) 12.25.

(1 mark)

- 38.** Because a new medical procedure has been shown to be effective in early detection of an illness, a medical screening of the population is proposed. The probability that the test correctly identifies some one with the illness as positive is 0.99, and the probability that the test correctly identifies someone without the illness as negative is 0.95. The incidence of illness in the general population is 0.0001. You take the test, and the result is positive. What is the probability that you have the illness? (Round off your answer upto the third decimal).
- (a) 0.002 (b) 0.005 (c) 0.010 (d) 0.020
(e) 0.050.

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- 39.** Two computers **A** and **B** are to be marketed. A salesman who is assigned the job of selling these has the chances of 60 percent and 40 percent respectively to get success. The two computers may be sold independently given that the salesman is able to sell at least one computer. What is the probability that computer **A** has been sold? (Round off your answer up to the second decimal).
- (a) 0.40 (b) 0.64 (c) 0.79 (d) 0.90
(e) 0.24.

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(1 mark)

- 40.** A consignment of 15 CD players contains 4 defectives. The CD players are selected at random, one by one,

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without replacement and then examined. What is the probability that the ninth one examined is the last defective?

- (a) 0.041 (b) 0.143 (c) 0.005 (d) 0.287 (e) 0.713.

(2 marks)

41.

If $P(A \text{ and } B) = \frac{1}{2}$, $P(\bar{A} \text{ and } \bar{B}) = \frac{1}{3}$ and $P(A) = P(B) = p$, then what is the value of p ? $\bar{A}$ and $\bar{B}$ are complements of events A and B respectively.

- (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{7}{12}$ (e) $\frac{1}{5}$.

(1 mark)

- • 42. Two fair dice are thrown. What is the probability that one of them gives an even number less than 5, and the other one gives an odd number less than 4?

- (a) $\frac{1}{9}$ (b) $\frac{2}{9}$ (c) $\frac{1}{3}$ (d) $\frac{4}{9}$ (e) $\frac{5}{9}$.

(1 mark)

43.

One shot is fired from each of three guns. G_1 , G_2 and G_3 denote the events that the target is hit by the first, second and third guns respectively. $P(G_1) = 0.5$, $P(G_2) = 0.6$ and $P(G_3) = 0.8$ and, G_1 , G_2 and G_3 are independent of each other. What is the probability of registering exactly one hit?

- (a) 0.22 (b) 0.24 (c) 0.26 (d) 0.28 (e) 0.30.

(1 mark)

44.

Sixty percent of the employees of a company are college

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graduates and out of these, ten percent are in sales. Of the employees who did not graduate from college, eighty percent are in sales. What is the probability that an employee selected at random is neither in sales nor a college graduate?

- (a) 0.04 (b) 0.06 (c) 0.08 (d) 0.10 (e) 0.12.

(2 marks)

45. If the letters of the word **VICTORY** are arranged in a row at random, the probability that no two vowels may come together is

- (a) $\frac{2}{7}$ (b) $\frac{3}{7}$ (c) $\frac{4}{7}$ (d) $\frac{5}{7}$ (e) $\frac{6}{7}$.

(1 mark)

46. Find the sum of **n** terms of the series:

$\log a + \log \frac{a^2}{b} + \log \frac{a^3}{b^2} + \log \frac{a^4}{b^3} + \dots$ to **n** terms.

- (a) $(n/2)[n\log(ab) - \log(a/b)]$ (b) $(n/2)[\log(ab) - n\log(a/b)]$
 (c) $(n/2)[n\log(ab) + \log(a/b)]$ (d) $(n/2)[\log(ab) + n\log(a/b)]$
 (e) $(n/2)[\log(a/b) - n\log(ab)].$

(1 mark)

47. The first term of an A.P. is **a**, second term is **b** and the last term is **c**, then the sum of all the terms of the series is

- (a) $\frac{(a+c)(b+c+2a)}{2(b+a)}$ (b) $\frac{(a+c)(b+c-2a)}{2(b-a)}$ (c) $\frac{(a-c)(b+c-2a)}{2(b-a)}$
 (d) $\frac{(a+c)(2a-b+c)}{2(b-a)}$ (e) Zero.

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48. If the ratio of the sums of m terms and n terms of an A.P. is $m^2 : n^2$, then the ratio of its m th and n th terms will be

- (a) $(2m + 1) : (2n + 1)$ (b) $(2m - 1) : (2n + 1)$
(c) $(2m - 1) : (2n - 1)$
(d) $(2m + 1) : (2n - 1)$ (e) $(m + 1) : (n + 1)$.

(1 mark)

49. The ratio between the sum of n terms of two A.P.'s is $(3n + 8) : (7n + 15)$. Find the ratio between their twelfth terms.

- (a) $6 : 17$ (b) $7 : 15$ (c) $8 : 17$ (d) $7 : 16$
(e) $9 : 19$.

(2 marks)

50. If the sums of p , q and r terms of an A.P. are a , b and c respectively, then find the value of the expression:

$$\frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q)$$

- (a) $\frac{abc}{pqr}$ (b) $\frac{a+b+c}{p+q+r}$ (c) $ap + bq + cr$
(d) $abc(p+q+r)$
(e) Zero.

(2 marks)

51. In an A.P. j times the j^{th} term is equal to k times the k^{th} term. The first term is a and common difference is d . What is the $(j+k)^{\text{th}}$ term of the A.P.?

- (a) 0 (b) $(j+k)$ (c) $(j+k-1)d$
(d) a
(e) d .

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52.

$$\frac{1}{2}x + \frac{1}{2}y - \frac{1}{6}z = 5$$

$$\frac{1}{3}x + \frac{1}{4}y + \frac{1}{2}z = 10$$

$$\frac{1}{6}x + \frac{1}{4}y + \frac{1}{3}z = 7$$

The value of **z** in the above simultaneous equations would be

- (a) 6 (b) 8 (c) 12 (d) 15 (e) 18.

(1 mark)

53.

$$x = \frac{\sqrt[q]{m^p}}{\sqrt[p]{n^q}}, \quad y = \frac{\sqrt[p]{n^{(q-1)}}}{\sqrt[q]{m^3}} \quad \text{and} \quad z = \sqrt[q]{m^{(2-p)}} \sqrt[p]{n^2}$$

The value of **xyz** would be

- (a) $\frac{\sqrt[q]{m}}{n}$ (b) $\frac{m}{\sqrt[p]{n}}$ (c) $\frac{\sqrt[p]{n}}{\sqrt[q]{m}}$ (d) $\frac{\sqrt[q]{m}}{\sqrt[p]{n}}$
(e) $\frac{n}{m}$.

(1 mark)

54.

The sums of first **n**, **2n** and **3n** terms of an A.P. are S_1 , S_2 and S_3 respectively. Which of the following equations would be **true**?

- (a) $S_3 = 4(S_2 - S_1)$ (b) $S_3 = 2(S_2 - S_1)$
(c) $S_2 = S_3 + S_1$
(d) $S_1 = S_3 - S_2$ (e) $S_3 = 3(S_2 - S_1)$.

(2 marks)

55.

The sum of four numbers of a G.P. is 85 and the product of the same is 4096. The highest of the three numbers is

- (a) 56 (b) 64 (c) 81 (d) 128
(e) 144.

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56. If the m^{th} term of an H.P. is n and the n^{th} term is m , then the $(m + n)^{\text{th}}$ term of the H.P. would be

(a) $\frac{m}{n}$ (b) $\frac{n}{m}$ (c) $\frac{m}{(m+n)}$ (d) $\frac{mn}{(m+n)}$ (e) $\frac{(m-n)}{(m+n)}$.

(1 mark)

57. If ${}^nC_6 : {}^{n-3}C_3 = 33:4$. The value of n would be

(a) 8 (b) 9 (c) 10 (d) 11 (e) 12.

(2 marks)

58. The number of words that can be formed by rearranging the letters of the word **CORRESPONDENCE** is

(a) $14!$ (b) $\frac{14!}{(2!)^3 3!}$ (c) $\frac{14!}{(2!)^4 3!}$ (d) $\frac{14!}{(2!)^2 3!}$ (e) $\frac{14!}{(2!)(3!)}$.

(1 mark)

59. S is the sum and P is the product of the first n terms in a G.P. R is the sum of the reciprocals of the first n terms of the G.P.

$S^n =$

(a) P^2 (b) R^n (c) P^n (d) PR^n (e) $P^2 R^n$.

(2 marks)

60. ${}^{n+1}C_r + {}^{n+1}C_{r-1} =$

(a) ${}^{n+1}P_r$ (b) ${}^{n+1}P_{r-1}$ (c) $\frac{{}^{n+2}P_r}{(r-1)!}$ (d) $\frac{{}^{n+2}P_r}{r!}$

$$(e) \frac{{}^{n+2}P_r}{(r+1)!}.$$

(2 marks)

61.

$$x = \frac{3^{\frac{4}{9}} \times 7^{\frac{2}{7}}}{5^{\frac{2}{3}}}, \quad y = \frac{5^{\frac{4}{9}} \times 3^{\frac{1}{3}}}{7^{\frac{2}{7}}} \quad \text{and} \quad z = \frac{3^{\frac{2}{9}}}{5^{\frac{2}{9}} \times 7^{\frac{6}{7}}}$$

The value of xyz is

- (a) $\frac{5}{21}$ (b) $\frac{7}{15}$ (c) $\frac{3}{35}$ (d) $\frac{15}{7}$
 (e) $\frac{21}{5}$.

(1 mark)

62.

$$x = \frac{\sqrt[5]{m^3}}{\sqrt{n}}, \quad y = \frac{\sqrt[3]{n}}{\sqrt[5]{m^2}} \quad \text{and} \quad z = \frac{\sqrt[5]{m^4}}{\sqrt[6]{n^5}}$$

The expression xyz is equal to

- (a) $\frac{\sqrt[5]{m}}{n}$ (b) $\frac{m}{\sqrt[3]{n}}$ (c) $\frac{m}{n}$ (d) $\frac{\sqrt[5]{m}}{\sqrt[3]{n}}$
 (e) $\frac{n}{m}$.

(1 mark)

63.

The arithmetic mean of two numbers exceeds their geometric mean by 15 and their harmonic mean by 27. The lowest of the two numbers is

- (a) 20 (b) 25 (c) 30 (d) 40
 (e) 50.

(2 marks)

64.

If $Z = {}^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7$ then the value of Z is

- (a) 4 (b) 3 (c) 2 (d) 1
 (e) 0.

(1 mark)

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65. How many committees of five members each can be formed from 8 official and 4 non-official members when these committees would require at least 2 non-official members? ≤
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≥
- (a) 112 (b) 336 (c) 448 (d) 456
(e) 480.
- (1 mark)
66. $81^{\frac{1}{\log_5 3}} + 27^{\log_9 36} + 3^{\frac{4}{\log_7 9}} =$ ≤
Ans
wer
≥
- (a) 216 (b) 625 (c) 720 (d) 890
(e) 1184.
- (1 mark)
67. If $\log_{12} 18 = a$, and $\log_{24} 54 = b$, the value of $ab + 5(a - b)$ would be ≤
Ans
wer
≥
- (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$
(e) 1.
- (2 marks)
68. If $\log_3 2$, $\log_3 (2^x - 5)$ and $\log_3 (2^x - 7/2)$ are in arithmetic progression, the value of x would be ≤
Ans
wer
≥
- (a) 9 or 10 (b) 7 or 8 (c) 6 or 7 (d) 4 or 5
(e) 2 or 3.
- (2 marks)
69. If $\frac{\log a}{(b - c)} = \frac{\log b}{(c - a)} = \frac{\log c}{(a - b)}$ then the value of $a^a b^b c^c$ would be ≤
Ans
wer
≥
- (a) a (b) b (c) c
(d) 1 (e) 0.
- (1 mark)
70. If the m th term of an A.P. is n and the n th term is m , then its $(m + n)$ th term will be ≤
Ans
wer
≥
- (a) $m + n$ (b) $m - n$ (c) $m \times n$ (d) $n - m$
(e) zero.

(1 mark)

71. The ratio of the sum of first three terms of a G.P. and the sum of the first six terms of the same G.P is 125:152. The common ratio of the series is

(a) $\frac{1}{3}$ (b) $\frac{1}{5}$ (c) $\frac{3}{5}$ (d) $\frac{5}{3}$
(e) $\frac{2}{3}$.

(2 marks)

72. Differentiation of $a^{\frac{1}{bx-c}}$ with respect to x is

(a) $\frac{ba^{\frac{1}{bx-c}} \times (bx-c)^2}{\ln a}$ (b) $\frac{ba^{\frac{1}{bx-c}} \times \ln a}{(bx-c)^2}$ (c) $-\frac{ba^{\frac{1}{bx-c}} \times (bx-c)^2}{\ln a}$
(d) $-\frac{ba^{\frac{1}{bx-c}} \times \ln a}{(bx-c)^2}$ (e) $ba^{\frac{1}{bx-c}} \times (bx-c)^2 \times \ln a$.

(1 mark)

73. If $\sqrt{x+y} + \sqrt{y-x} = c$, then $\frac{d^2y}{dx^2} =$

(a) $2/c$ (b) $-2/c$ (c) $2/c^2$ (d) 2
(e) $-2/c^2$.

(2 marks)

74. The limit of the function $f(x) = \frac{x^2 - 4x + 3}{x^2 - 1}$ at $x = 1$

(a) 0 (b) 1 (c) -1 (d) 2
(e) -2 .

(1 mark)

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75.

The limit of the function $f(x) = \frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3}$ at $x = 1$

- (a) 0 (b) $\frac{1}{5}$ (c) $\frac{1}{10}$ (d) $-\frac{1}{10}$
 (e) $-\frac{1}{5}$.

(1 mark)

76.

Differentiation of $\ln(a-bx)$ with respect to x is

- (a) $\frac{a}{(a-bx)}$ (b) $\frac{b}{(bx-a)}$ (c) $\frac{-a}{(a-bx)}$ (d) $\frac{-b}{(bx-a)}$ (e) $-e^{(bx-a)}$.

(1 mark)

77.

Differentiation of $\ln\left(\frac{1}{x+a}\right)$ with respect to x is

- (a) $(x+a)$ (b) $\frac{1}{(x+a)}$ (c) $-(x+a)$ (d) $-\frac{1}{(x+a)}$
 (e) $-\frac{1}{(x+a)^2}$.

(1 mark)

78.

Differentiation of $e^{\frac{1}{x+a}}$ with respect to x is

- (a) $\frac{(x+a)\sqrt{e}}{(x+a)^2}$ (b) $-\frac{(x+a)\sqrt{e}}{(x+a)}$ (c) $-\frac{(x+a)\sqrt{e}}{(x+a)^2}$ (d) $\frac{(x+a)\sqrt{e}}{(x+a)}$
 (e) $\frac{e}{(x+a)}$.

(1 mark)

79.

Differentiation of $\frac{1}{\sqrt{x-a}}$ with respect to x is

- (a) $\frac{1}{2}(x-a)^{-\frac{3}{2}}$ (b) $\frac{1}{2(x-a)^{-\frac{3}{2}}}$ (c) $\frac{1}{2\sqrt{x-a}}$ (d)

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$$-\frac{1}{2\sqrt{x-a}}$$

$$-\frac{1}{2(x-a)^{\frac{3}{2}}}$$

(e)

(1 mark)

80.

A firm has the following revenue function for its product:

$$R = 8Q$$

Where, **R** = Revenue earned in a day

Q = Number of units of the product produced and sold in a day

The total cost of producing and selling the product on any day is given by the following function:

$$C = 1,26,000 + 60\left(\frac{Q}{900}\right)^2$$

Where, **C** = The total cost of producing and selling the product in a day

Q = Number of units of the product produced and sold in a day

What is the number of units of the product that must be produced and sold by the firm in a day in order to maximize its daily profit?

- (a) 6,750 units (b) 12,500 units (c) 40,000 units
(d) 54,000 units (e) 1,26,000 units.

(2 marks)

81.

$$\lim_{x \rightarrow \infty} \frac{x^2 - 4}{2 + x - 4x^2} =$$

- (a) -2 (b) -1/4 (c) 1/2 (d) 1 (e) ∞.

(1 mark)

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82.

$$\text{Let } f(x) = \begin{cases} 3^{-x} - 1, & \text{for } -1 \leq x < 0 \\ \frac{x}{x^2 - 2}, & \text{for } 0 \leq x < 3 \\ 2^x - 11, & \text{for } 3 \leq x \leq 6 \end{cases}$$

The value of $f(4)$ is

- (a) $-\frac{80}{81}$ (b) $-\frac{82}{81}$ (c) $\frac{2}{7}$ (d) 5
(e) -19.

(1 mark)

83

$$\text{If } y = f(x) = \frac{x+2}{x-1} \text{ then } f(y) =$$

- (a) $-x$ (b) x^2 (c) x (d) $(1-x)$
(e) $(x+1)$.

(1 mark)

**END OF THE
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Suggested Answers

Quantitative Methods – I (131): January 2005

1. Answer :

Reason : If there are n items in data then the average deviation from zero can be written as

$$\begin{aligned} & \frac{(x_1 - 0) + (x_2 - 0) + \dots + (x_n - 0)}{n} \\ &= \frac{\sum_{i=1}^n x_i - 0 \times n}{n} = \frac{\sum_{i=1}^n x_i}{n} = \text{Mean.} \end{aligned}$$

2. Answer :

Reason : If every item in a data set is increased by a constant C , then the arithmetic mean of the resulting data set will be equal to the mean of the original data set plus C , i.e., $C + \text{mean of the original data set}$.

3. Answer :

Reason : Co-efficient of variation is a relative measure of dispersion based upon standard deviation and mean.

$$\text{Co-efficient of variation} = \frac{\text{Standard Deviation}}{\text{Mean}} \times 100\%$$

4. Answer :

Reason : Standard deviation is independent of change of origin i.e., it remains unchanged even if all the items in the data set are increased or decreased by the same quantity.

5. Answer :

Reason : The appropriate mean for a set of ratios using the denominators of the ratios as weights is weighted arithmetic mean.

6. Answer :

Reason : Standard deviations of two or more data sets can be mathematically combined

Arithmetic means of two or more data sets can be mathematically combined

Geometric means of two or more data sets can be mathematically combined

Harmonic means of two or more data sets can be mathematically combined.

Medians of two or more data sets cannot be mathematically combined

7. Answer :

Reason : The standard deviation of a data set is expressed in the same unit as the observations in the data set.

(b), (c), (d) and (e) are incorrect with regard to the standard deviation.

8. Answer :

Reason : Coefficient of variation = $(\text{standard deviation} / \text{mean}) \times 100$

A standard deviation equal to zero implies that there is no deviation in the data set. The same will be reflected by the c.v. provided mean is not equal to zero.

Even when the standard deviation is 1 the c.v. can be meaningfully used for comparison of variability provided mean is not equal to zero.

Hence it cannot be meaningfully used for comparison of variability when mean of one or more data sets is zero.

When the mean is equal to 1, the c.v. can be meaningfully used for comparison of variability.

When the mean and standard deviation are equal for one or more sets of data, the c.v. can be meaningfully used for comparison of variability.

9. Answer :

Reason : If there is no dispersion in a data set then all the mathematical and positional averages are equal. TOP
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Hence b. is false.

Hence c. is false.

Hence d. is false.

Hence e. is false.

10. Answer :

Reason : The tallest rectangle in a histogram represents the modal class. TOP
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In a symmetrical distribution mean, median and mode are equal.

The median can not be mathematically manipulated; hence medians of two sets of data can not be combined.

The median can be determined graphically.

The mode is not uniquely determined when more than one observations have the highest frequency.

11. Answer :

Reason : The constraint in equations specify the consumption of resources and the amount of resources available. TOP
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12. Answer :

Reason : (a), (b), (d) and (e) are all assumptions underlying linear programming. However, (c) is not an assumption because linear programming assumes that the amount of resources consumed by each unit of the decision variables is certain. TOP
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13. Answer :

Reason : In the graphical method of solving LP problems the decision variables are represented by the TOP
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horizontal and vertical axes.

This method can be applied when there are two decision variables.

If the optimal solution exists then it occurs at one of the corner points.

The optimal solution to the LPP can not occur at a point outside the feasible region because a point outside the feasible region means that the constraints are violated.

The feasible region is determined by the structural constraints as well as the non-negativity constraints.

14. Answer :

Reason : At the optimal solution the slack variables need not be equal to zero.

The constraints which contain the $\leq$ sign are converted into equations by adding slack variables.

There can be more than one feasible solution to a LPP.

The slack variables make zero contribution towards profit.

The slack variables can only assume any non-negative value.

15. Answer :

Reason : (a) and (b) are not the indicators of multioptimality.

In the graphical method of solving a LPP the situation of multioptimality arises when the objective function is parallel to one of the edges of the feasible region which is in the direction of the optimal movement of the objective function.

(d) and (e) are not correct because they only assume non-negative values.

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16. Answer :

Reason : The classical approach to probability assumes that the outcomes are equally likely.

In the relative frequency approach to probability the probability of an event is determined after performing the experiment large number times.

In the classical approach to probability the probability of an event is determined before performing the experiment.

The classical approach to probability assumes that all possible outcomes of the experiment are known.

The classical approach can be used to find out the probability of mutually exclusive events.

17. Answer :

Reason : If events A and B are mutually exclusive then $P(A \text{ and } B) = 0$ because A and B can not occur at the same time.

Mutual exclusiveness of two events A and B does not imply that both of them have zero probabilities of occurrence.

Mutual exclusiveness does not mean that the probability of any one of them occurring is either 0 or 1.

18. Answer :

Reason : The probability of the occurrence of an event is expressed as a number which lies between 0 and 1.

19. Answer :

Reason : (a) implies that the function is $f(x)$ is increasing at $x = c$.

(b) implies that the function is $f(x)$ is decreasing at $x = c$.

(c) implies that a function is changing at a constant

rate.

(d) is true when the function has a relative minima at $x = c$.

(e) is true when the function has a relative maxima at $x = c$.

20. Answer :

Reason : The derivative of a function indicates the rate of change of the function. [TOP](#)

The slope of the tangent to a function at a point is equal to the derivative of the function at that point.

The derivative of a function can be said to be a function of the independent variable if the expression of the derivative contains the independent variable.

The derivative of a linear function is the slope of the linear function, which is a constant value for all values of the dependent variable.

If the derivative of any function at a point is negative then it indicates that the function is decreasing at that point.

21. Answer :

Reason : For a function $f(x)$ the first derivative is positive and the second derivative is negative at $x = a$. This means that $f(x)$ is increasing at a decreasing rate at $x = a$. For a function $f(x)$ to be minimum or maximum the first derivative should be equal to zero. [TOP](#)

22. Answer :

Reason : If the first derivative of $f(x)$ is a constant for all values of x then it indicates that the function changes at a constant rate. [TOP](#)

A function $f(x)$ is said to be monotonically decreasing if the first derivative of $f(x)$ is negative for all values of x .

A function $f(x)$ is said to be monotonically increasing if the first derivative of $f(x)$ is positive for all values of x .

If the first derivative of $f(x)$ is zero for all values of x then it indicates that the function has a constant value.

This is not the condition for a monotonically increasing function.

23. Answer :

Reason : For any function $f(x)$ the limit of $f(x)$ as x approaches a value a is a value L , such that $f(x)$ approaches L as x approaches a .

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24. Answer :

Reason : A function is a rule or correspondence which always associates to each number x in a set A , a unique number $f(x)$ in a set B .

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25. Answer :

Reason : a. This interval is bounded on the lower side but unbounded on the upper side.

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b. This interval is bounded on the upper side but unbounded on the lower side.

c. This interval is bounded on the lower side but unbounded on the upper side.

d. This interval is bounded on the upper side but unbounded on the lower side.

e. This interval is bounded on the upper as well as lower sides.

26. Answer :

Reason : The reciprocal of the terms in a harmonic progression are in arithmetic progression. And vice versa. Therefore the reciprocals of a H.P. cannot be in H.P. There is no such connection between the H.P.

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and the G.P. If the terms in the corresponding A.P. are in the increasing order then the terms of the H.P. will be in the decreasing order and vice versa. Hence there is no reason why terms of a H.P. will always be in increasing or decreasing order.

27. Answer :

Reason : The consecutive terms of the G.P. will be in increasing order if the first term in a geometric progression is greater than one and the common ratio is more than 1.

The consecutive terms of the G.P. will be in decreasing order if the first term in a geometric progression is greater than one and the common ratio is less than 1.

If the first term in a geometric progression is greater than one and the common ratio is less than 1, then the consecutive terms will not be the same.

All the consecutive terms will be less than 1 if the first term as well as the common ratio is less than 1.

All the consecutive terms will be greater than 1 if the first term as well as the common ratio is more than 1. However a decreasing G.P. may still have all the terms greater than 1; this depends upon the number of terms in the G.P.

28. Answer :

Reason : If all the terms of an arithmetic progression are multiplied by a constant the resulting terms will always form an arithmetic progression with the first term multiplied by the constant as well as the common difference multiplied by the constant. The resulting series will neither be in a geometric series or a harmonic series because nature of the resulting terms will not satisfy their requirements.

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29. Answer :

Reason : The set of whole numbers includes the set of natural numbers. [TOP](#)

The set of whole numbers does not include the set of rational numbers.

The set of whole numbers does not include the set of negative numbers.

The set of whole numbers does not include the set of complex numbers.

The set of whole numbers does not include the set of irrational numbers.

30. Answer :

Reason : a. This is not a property of the real numbers [TOP](#)

b. According to the inverse property of addition for every real number there exists another number such that the sum of the two real numbers is equal to 0.

c. This is the identity property of addition.

d. This is the inverse property of multiplication.

e. This is the identity property of multiplication.

31. Answer :

Reason : In a positively skewed distribution the majority of the observations are concentrated towards the lower end of the scale. In a negatively skewed distribution majority of the observations are concentrated towards the higher end of the scale. In a skewed distribution the distribution of the data is not symmetrical. [TOP](#)

32. Answer :

Reason : Range (= Highest value – Lowest value) is based only on two observations in a data set. [TOP](#)

Arithmetic mean, harmonic mean, mean absolute

deviation and standard deviation are based on all the observations.

33. Answer :

Reason : Range is based on the two extreme values in the data set. Hence it is affected by the extreme values in the data set.

Arithmetic mean is based on all the values. Hence this is also affected by the presence of extreme of values.

Standard deviation is based on all the values. Hence this is also affected by the presence of extreme of values.

Variance is based on all the values. Hence this is also affected by the presence of extreme of values.

Median is middle value of a data array. So it is not affected by the presence of the extreme values.

34. Answer :

Reason : If every observation in a data set is increased by a constant then the coefficient of variation of the resulting set of values will be less than the coefficient of variation of the original data set because the numerator remains the same whereas the denominator increases.

35. Answer :

Reason : If two events A and B are independent then, the conditional probability of event A given event B is equal to marginal probability of event A because the occurrence of event B does not influence the occurrence of event A.

36. Answer :

Reason :
$$S^2 = \frac{1}{n-1} [\sum x^2 - n\bar{X}^2] = \frac{1}{n-1} \left[\sum x^2 - \frac{(\sum x)^2}{n} \right]$$

$$\sum_{i=1}^{10} x_i = \sum_{i=1}^4 x_i + \sum_{i=5}^{10} x_i$$

$$\sum_{i=1}^4 x_i = 16 \times 4 = 64$$

$$\sum_{i=5}^{10} x_i = 11 \times 6 = 66$$

$$\therefore \sum_{i=1}^{10} x_i = 64 + 66 = 130$$

$$\sum_{i=1}^{10} x_i^2 = 1726 \quad (\text{given})$$

$$\therefore n = 10$$

$$\therefore S^2 = \frac{1}{9} \left[1726 - \frac{(130)^2}{10} \right] = 4$$

$$\therefore S = \sqrt{4} = 2$$

37. Answer :

Reason : $N_A = 15 \quad \mu_A = 20 \quad \sigma_A = 4$

$N_B = 25 \quad \mu_B = 16 \quad \sigma_B = 2$

$$\mu = \frac{N_A \mu_A + N_B \mu_B}{N_A + N_B} = \frac{(15 \times 20) + (25 \times 16)}{15 + 25} = \frac{300 + 400}{40} = 17.5.$$

$$d_A = \mu_A - \mu = 20 - 17.5 = 2.5$$

$$d_B = \mu_B - \mu = 16 - 17.5 = -1.5$$

$$\sigma_{12} = \left[\frac{N_A \sigma_A^2 + N_B \sigma_B^2 + N_A d_A^2 + N_B d_B^2}{N_A + N_B} \right]^{1/2}$$

$$= \left[\frac{(15 \times 4^2) + (25 \times 2^2) + (15 \times 2.5^2) + [25(-1.5^2)]}{15 + 25} \right]^{1/2}$$

$$= \left[\frac{240 + 100 + 93.75 + 56.25}{40} \right]^{1/2} = 3.50.$$

38. Answer : (a)

Reason : Let PI = the event that the test is positive and NI = the event that the test is negative.

[TOP](#)

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Also, let D denote the event that subject has the illness and D^c the complementary event that the subject does not have the illness. The following information has been provided in this problem:

$$P(\text{PI} \mid D) = 0.99 \text{ and therefore } P(\text{NI} \mid D) = 0.01$$

$$P(\text{NI} \mid D^c) = 0.95 \text{ and therefore } P(\text{PI} \mid D^c) = 0.05$$

$$P(D) = 0.0001 \text{ and therefore } P(D^c) = 0.9999$$

Our goal is to find $P(D \mid \text{PI})$. By the Bayes' theorem

$$\begin{aligned} P(D \mid \text{PI}) &= \frac{P(D \text{ and PI})}{P(\text{PI})} = \frac{\{P(\text{PI} \mid D)P(D)\}}{\{P(\text{PI} \mid D)P(D) + P(\text{PI} \mid D^c)P(D^c)\}} \\ &= \frac{\{0.99 \times 0.0001\}}{\{0.99 \times 0.0001 + 0.05 \times 0.9999\}} = 0.002 \text{ (approx.)} \end{aligned}$$

39. Answer : (c)

Reason : Let the events A and B denote that the computers A and B respectively be marketed. As per the problem, $P(A) = 0.60$ and $P(B) = 0.40$ and hence, $P(\bar{A}) = 0.40$ and $P(\bar{B}) = 0.60$

The probability of selling at least one computer is given by $P(A \cup B)$. Hence, the required probability may be obtained as: $P\{A/(A \cup B)\}$. Therefore, from the multiplication theorem of probability, we get,

$$\begin{aligned} P\{A/(A \cup B)\} &= \frac{P[A \text{ and } (A \cup B)]}{P(A \cup B)} = \frac{P(A)}{1 - P(\bar{A} \text{ and } \bar{B})} = \frac{0.6}{1 - 0.6 \times 0.4} \\ &= \frac{0.6}{1 - 0.24} = \frac{0.6}{0.76} = 0.79 \text{ (approx.)} \end{aligned}$$

40. Answer : (a)

Reason : Let A indicates the event that exactly 3 defectives are found after checking 8 CDs and B denotes that the 9th piece has been examined and has been found defective. Therefore, by the classical definition of probability we get,

$$P(A) = \frac{{}^4C_3 \times {}^{11}C_5}{{}^{15}C_8} = \frac{56}{195}$$

Now, $P(B/A)$ = The probability that the 9th CD is a defective one provided that there were 3 defective CDs found in the examination of the first 8 CDs =

$$\frac{4-3}{15-8} = \frac{1}{7}$$

Hence, the required probability is given by: $P(A \text{ and } B) = P(A) \times P(B/A) = \frac{56}{195} \times \frac{1}{7} = \frac{8}{195}$

$$= 0.041 \text{ (approx.)}$$

41. Answer : (d)

Reason : We have, $P(A \text{ and } B) = 1/2$ and $P(\bar{A} \text{ and } \bar{B}) = 1/3 = 1 - P(A \text{ or } B)$

$$\text{or, } P(A) + P(B) - P(A \text{ and } B) = 1 - \frac{1}{3} = \frac{2}{3} \quad \therefore P(A) + P(B) = P(A \text{ or } B) + P(A \text{ and } B) = \frac{2}{3} + \frac{1}{2} = \frac{7}{6}$$

$$\text{Now, } P(A) = P(B) = p. \text{ Therefore, } 2P = \frac{7}{6} \text{ or } P = \frac{7}{12}.$$

42. Answer : (b)

Reason : Probability that the first dice gives an even number less than 5 and the second dice gives an odd number

$$\text{less than 4} = \frac{2}{6} \times \frac{2}{6} = \frac{1}{9}$$

Probability that the second dice gives an even number less than 5 and the first dice gives an odd number less

$$\text{than 4} = \frac{2}{6} \times \frac{2}{6} = \frac{1}{9}$$

Since these events are mutually exclusive, the probability that one of the dice gives an even number less than 5 and the other one gives an odd number less

$$\text{than 4} = \frac{1}{9} + \frac{1}{9} = \frac{2}{9}$$

43. Answer : (c)

Reason : We are given that $P(G_1) = 0.5$, $P(G_2) = 0.6$ and $P(G_3) = 0.8$ and hence their complementary events are given by:

$$P(\bar{G}_1) = 0.5, P(\bar{G}_2) = 0.4 \text{ and } P(\bar{G}_3) = 0.2.$$

The event of exactly one hit may be the combinations of the following mutually exclusive events:

(i) G_1 and $\bar{G}_2$ and $\bar{G}_3$ happens (ii) $\bar{G}_1$ and G_2 and $\bar{G}_3$ happens
or

(iii) $\bar{G}_1$ and $\bar{G}_2$ and G_3 happens

Therefore from the addition rule of probability, the required probability may be obtained as:

$$\begin{aligned} P &= P(G_1 \text{ and } \bar{G}_2 \text{ and } \bar{G}_3) + P(\bar{G}_1 \text{ and } G_2 \text{ and } \bar{G}_3) + P(\bar{G}_1 \text{ and } \bar{G}_2 \text{ and } G_3) \\ &= P(G_1)P(\bar{G}_2)P(\bar{G}_3) + P(\bar{G}_1)P(G_2)P(\bar{G}_3) + P(\bar{G}_1)P(\bar{G}_2)P(G_3) \\ &= 0.5 \times 0.4 \times 0.2 + 0.5 \times 0.6 \times 0.2 + 0.5 \times 0.4 \times 0.8 = \\ &0.04 + 0.06 + 0.16 = 0.26 \end{aligned}$$

44. Answer : (c)

Reason : Let A be the event that an employee is a college graduate and B be the event that an employee is in sales. Hence, $P(A) = 0.60$, $P(B/A) = 0.10$ and $P(B/\bar{A}) = 0.80$.

$$\begin{aligned} P(B) &= P(A \text{ and } B) + P(\bar{A} \text{ and } B) = P(A) \times P(B/A) + P(\bar{A}) \times P(B/\bar{A}) \\ &= 0.60 \times 0.10 + (1 - 0.60) \times 0.80 = 0.06 + 0.32 = \\ &0.38 \end{aligned}$$

Now, the probability that a employee selected at random is neither in sales nor a college graduate is given by: $P(\bar{A} \text{ and } \bar{B}) = 1 - P(A \text{ or } B) = 1 - \{P(A) + P(B) - P(A \text{ and } B)\}$

$$= 1 - \{0.60 + 0.38 - 0.6 \times 0.10\} = 1 - 0.92 = 0.08.$$

45. Answer :

Reason : The total number of ways the letters of the word [TOP](#)
VICTORY can be arranged is $n(S) = 7!$ [≥](#)

The number of ways two vowels always come together is $= 2!.6!$

Therefore total number of ways when the vowels do not come together is $n(E) = 7! - 2!.6! = 5.6!$

Therefore the probability of favorable event $= \frac{n(E)}{n(S)} = \frac{5}{7}$

46. Answer :

Reason : We find that the given series is in A.P. with first [TOP](#)
term as $\log a$ and with a common difference of $\log$ [≥](#)
 (a/b)

Now sum of the n terms of the series is

$$\begin{aligned} S_n &= (n/2) [2 \log a + (n-1) \log (a/b)] \\ &= (n/2) [\log a^2 + n \log (a/b) - \log (a/b)] \\ &= (n/2) [\log ab + n \log (a/b)] \end{aligned}$$

47. Answer :

Reason : Given that $T_1 = a$ and $T_2 = a + d = b \therefore d = b - a$ [TOP](#)
[≥](#)

$$T_n = a + (n-1)d = c$$

$$n-1 = \frac{c-a}{d} = \frac{c-a}{b-a}$$

$$n = \frac{c-a}{b-a} + 1 = \frac{c+b-2a}{b-a} \dots\dots(i)$$

$$S_n = (n/2) [a + T_n] = \frac{(c+b-2a)}{2(b-a)}(a+c) \dots\dots(\text{using the relation (i)})$$

48. Answer :

Reason : Given that $\frac{S_m}{S_n} = \frac{m^2}{n^2}$ [TOP](#)
[≥](#)

$$\therefore \frac{(m/2)[2a+(m-1)d]}{(n/2)[2a+(n-1)d]} = \frac{m^2}{n^2}$$

$$\therefore \frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$$

Simplifying the above relation we get $d = 2a$

$$\text{Now } \frac{T_m}{T_n} = \frac{a + (m-1)d}{a + (n-1)d} = \frac{a + (m-1)2a}{a + (n-1)2a} = \frac{2m-1}{2n-1}$$

49. Answer :

Reason : Given that $\frac{S_n}{S'_n} = \frac{(n/2)[2a + (n-1)d]}{(n/2)[2a' + (n-1)d']} = \frac{3n+8}{7n+15}$

$$\text{Or, } \frac{a + [(n-1)/2]d}{a' + [(n-1)/2]d'} = \frac{3n+8}{7n+15} \dots\dots\dots(i)$$

$$\text{We have to find } \frac{T_{12}}{T'_{12}} = \frac{a+11d}{a'+11d'}$$

In an A.P., kth term = $a + (k - 1)d$

$$\therefore 12\text{th term} = a + 11d$$

$\therefore$ We can substitute the coefficients of d and d' respectively in the numerator and denominator of LHS of equation (i), by 11, for simplifying equation (i).

Substituting $(n-1)/2 = 11$ [i.e., $n = 23$] in equation (i), we get

$$\frac{T_{12}}{T'_{12}} = \frac{a+11d}{a'+11d'} = \frac{3 \times 23 + 8}{7 \times 23 + 15} = \frac{77}{176} = \frac{7}{16}.$$

50. Answer :

Reason : Let the first term of the A.P. be A and the common difference be D , then

$$S_p = a = (p/2)[2A + (p-1)D]$$

$$\therefore \frac{a}{p} = \left(A - \frac{D}{2}\right) + \frac{p}{2} \cdot D \dots\dots\dots(i)$$

$$\text{Similarly, } \frac{b}{q} = \left(A - \frac{D}{2}\right) + \frac{q}{2} \cdot D \dots\dots\dots(ii)$$

$$\text{And, } \frac{c}{r} = \left(A - \frac{D}{2}\right) + \frac{r}{2} \cdot D \dots\dots\dots(iii)$$

Multiplying equations (i), (ii) and (iii) with $(q - r)$, $(r - p)$ and $(p - q)$ respectively and adding them

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together, we get,

$$\begin{aligned} & \frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q) \\ &= \left(A - \frac{D}{2}\right)(q-r+r-p+p-q) + \frac{D}{2}[p(q-r) + q(r-p) + r(p-q)] \\ &= 0 \end{aligned}$$

51. Answer :

Reason : Let 'a' be the first term and 'd' be the common difference in the A.P.

$$\therefore \text{ jth term, } t_j = a + (j - 1) d$$

$$\text{ kth term, } t_k = a + (k - 1) d$$

$$\text{ Given: } j [a + (j - 1)d] = k [a + (k - 1) d]$$

$$\text{ or } ja + j(j - 1)d = ka + k(k - 1) d$$

$$\text{ or } a(j - k) = [k(k - 1) - j(j - 1)] d$$

$$\text{ or } a(j - k) = [k^2 - k - j^2 + j] d$$

$$\text{ or } a(j - k) = [k^2 - j^2 - (k - j)] d$$

$$\text{ or } a(j - k) = [(k - j)(k + j) - (k - j)] d$$

$$\text{ or } a(j - k) = [(k - j)(k + j - 1)] d$$

$$\text{ or } a = \frac{(k - j)}{(j - k)}(k + j - 1)d$$

$$\text{ or } a = -(j + k - 1) d \quad \dots\dots\dots (1)$$

$$(j + k)^{\text{th}} \text{ term, } t_{(j+k)} = a + (j + k - 1) d$$

From (1) above,

$$t_{(j+k)} = -(j + k - 1) d + (j + k - 1) d = 0$$

52. Answer :

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$$\frac{1}{2}x + \frac{1}{2}y - \frac{1}{6}z = 5 \dots\dots\dots(i)$$

$$\frac{1}{3}x + \frac{1}{4}y + \frac{1}{2}z = 10 \dots\dots\dots(ii)$$

$$\frac{1}{6}x + \frac{1}{4}y + \frac{1}{3}z = 7 \dots\dots\dots(iii)$$

By doing (ii) + (iii) - (i) we get

$$\left(\frac{1}{3} + \frac{1}{2} + \frac{1}{6}\right)z = (7 + 10 - 5)$$

Reason : Or, $z = 12$

53. Answer :

Reason :

$$\begin{aligned} xyz &= \frac{\sqrt[q]{m^p}}{\sqrt[p]{n^q}} \times \frac{\sqrt[p]{n^{(q-1)}}}{\sqrt[q]{m^3}} \times \sqrt[q]{m^{(2-p)}} \sqrt[p]{n^2} = \frac{m^{\frac{p}{q}}}{n^{\frac{q}{p}}} \times \frac{n^{\frac{q-1}{p}}}{m^{\frac{3}{q}}} \times m^{\frac{2-p}{q}} n^{\frac{2}{p}} \\ &= \frac{m^{\left(\frac{p}{q} - \frac{3}{q} + \frac{2-p}{q}\right)}}{n^{\left(\frac{q}{p} - \frac{q-1}{p} - \frac{2}{p}\right)}} \\ &= \frac{m^{\frac{(p-3-p+2)}{q}}}{n^{\frac{(q-q+1-2)}{p}}} \\ &= \frac{m^{-\frac{1}{q}}}{n^{-\frac{1}{p}}} = \frac{n^{\frac{1}{p}}}{m^{\frac{1}{q}}} = \frac{\sqrt[p]{n}}{\sqrt[q]{m}} \end{aligned}$$

54. Answer :

Reason :

$$\begin{aligned} S_1 &= \{2 \times a + (n-1) \times d\} \frac{n}{2} = na - \frac{nd}{2} + \frac{n^2 d}{2} \\ S_2 &= \{2 \times a + (2n-1) \times d\} \frac{2n}{2} = 2na - nd + 2n^2 d \\ S_3 &= \{2 \times a + (3n-1) \times d\} \frac{3n}{2} = 3na - \frac{3nd}{2} + \frac{9n^2 d}{2} \\ \text{Or, } S_3 &= 3 \times \left\{ \left(2na - nd + 2n^2 d \right) - \left(na - \frac{nd}{2} + \frac{n^2 d}{2} \right) \right\} = 3(S_2 - S_1) \end{aligned}$$

55. Answer :

Reason :

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Let the numbers be $\frac{m}{r^3}$, $\frac{m}{r}$, mr , mr^3 The product of

the four numbers is 4096

i.e. $m^4 = 4096 = 8^4$; So, $m = 8$

The sum of the numbers is $\frac{m}{r^3} + \frac{m}{r} + mr + mr^3 = 85$

$$\text{Or, } 8\left(\frac{1}{r^3} + \frac{1}{r} + r + r^3\right) = 85 \quad \text{Or, } 8\left(\frac{1}{r^3} + r^3\right) + 8\left(\frac{1}{r} + r\right) = 85$$

$$\text{Let } \left(\frac{1}{r} + r\right) \text{ be } z \quad \left(\frac{1}{r^3} + r^3\right) = \left(\frac{1}{r} + r\right)^3 - 3\left(\frac{1}{r} + r\right) = z^3 - 3z$$

$$\text{So, } 8(z^3 - 3z) + 8z = 85$$

$$\text{Or, } 8z^3 - 16z = 85 \quad \text{Or, } (2z - 5)(4z^2 + 10z + 17) = 0$$

The second factor has the imaginary roots so,

$$\text{the feasible root of } z = \frac{5}{2}$$

$$\left(\frac{1}{r} + r\right) = \frac{5}{2}$$

$$\text{Or, } 2r^2 + 2 = 5r \quad \text{Or, } 2r^2 - 5r + 2 = 0$$

$$\text{Or, } 2r(r - 2) - 1(r - 2) = 0$$

$$\text{Or, } (2r - 1)(r - 2) = 0 \quad \text{Or, } r = 2 \text{ or } \frac{1}{2}$$

So, the highest number is $8 \times 2^3 = 64$

56. Answer :

Reason : The m th term of the A.P. is $1/n$ and the n th term of the A.P is $1/m$.

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Let the first term of the AP be A and common difference be D

$$\text{So, } A + (m - 1)D = 1/n$$

$$\text{And, } A + (n - 1)D = 1/m$$

By subtracting these two equations we get

$$(m - n)D = 1/n - 1/m = (m - n)/mn$$

$$\text{Or, } D = 1/mn$$

So, $m+n$ th term would exceed m th term by nD

$$\text{So the } (m+n)\text{th term is } nD + 1/n = 1/n + 1/m = (m+n)/mn$$

So the corresponding term in H.P. would be $\frac{mn}{(m+n)}$

57. Answer :

Reason :

$${}^nC_6 : {}^{(n-3)}C_3 = 33 : 4$$

$$\text{Or, } \frac{{}^nC_6}{{}^{(n-3)}C_3} = \frac{33}{4}$$

$$\text{Or, } 4 \times {}^nC_6 = 33 \times {}^{(n-3)}C_3$$

$$\text{Or, } 4 \times \frac{n!}{6!(n-6)!} = 33 \times \frac{(n-3)!}{3!(n-3-3)!} = 33 \times \frac{(n-3)!}{3!(n-6)!}$$

$$\text{Or, } 4 \times \frac{(n-2)(n-1)n}{4 \times 5 \times 6} = 33$$

$$\text{Or, } (n-2)(n-1)n = 30 \times 33 = 3 \times 10 \times 3 \times 11 = 9 \times 10 \times 11 = (11-2)(11-1)(11)$$

So the value of $n = 11$

58. Answer :

Reason : In the word 'CORRESPONDENCE' the total number of letters are 14, C's are 2, O's are 2, R's are 2 E's are 3 and N's are 2.

$$\therefore \text{Total no. of ways} = \frac{14!}{(2!)^4 \cdot 3!}$$

59. Answer :

$$\text{Reason : } S = \frac{a(1-r^n)}{1-r}$$

$$P = a \times ar \times ar^2 \dots \dots \dots \times ar^{n-1} = a^n r^{1+2+\dots \dots \dots n-1}$$

$$= a^n r^{\frac{n(n-1)}{2}}$$

$$[1+2+\dots \dots n-1] = \frac{(n-1)}{2} [2 \times 1 + (n-2)1] = \frac{n-1}{2} [2+n-2] = \frac{n(n-1)}{2}$$

$$\therefore P^2 = a^{2n} r^{n(n-1)}$$

$$R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \dots \dots \dots \frac{1}{ar^{n-1}}$$

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$$\begin{aligned}
&= \frac{\frac{1}{a} \times \left[\left(\frac{1}{r} \right)^n - 1 \right]}{\frac{1}{r} - 1} = \frac{\frac{r}{a} \times \frac{1 - r^n}{(1 - r)r^n}}{\frac{1}{r} - 1} \\
&\therefore \frac{S}{R} = \frac{a(1 - r^n)}{(1 - r)} \times \frac{a(1 - r)r^n}{r(1 - r^n)} = a^2 r^{n-1} \\
&\therefore \frac{S^n}{R^n} = a^{2n} r^{n(n-1)} = P^2 \\
&\therefore S^n = P^2 R^n
\end{aligned}$$

60. Answer :

Reason : ${}^{n+1}C_r + {}^{n+1}C_{r-1}$

$$\begin{aligned}
&= \frac{(n+1)!}{r!(n-r+1)!} + \frac{(n+1)!}{(r-1)!(n+1-r+1)!} = \frac{(n+1)!}{r!(n-r+1)!} + \frac{(n+1)!}{(r-1)!(n-r+2)!} \\
&= \frac{(n+1)!}{(r-1)!(n-r+1)!} \left[\frac{1}{r} + \frac{1}{n-r+2} \right] \\
&= \frac{(n+1)!}{(r-1)!(n-r+1)!} \times \left[\frac{n-r+2+r}{r(n-r+2)} \right] \\
&= \frac{(n+1)!}{(r-1)!(n-r+1)!} \times \frac{(n+2)}{r(n-r+2)} \\
&= \frac{(n+1)! \times (n+2)}{[(r-1)! \times r][n(n-r+2)]} \\
&= \frac{(n+2)!}{r!(n+2-r)!} = {}^{n+2}C_r
\end{aligned}$$

$$\text{Now , } {}^{n+2}P_r = \frac{(n+2)!}{(n+2-r)!}$$

$$\therefore {}^{n+2}C_r = \frac{{}^{n+2}P_r}{r!}$$

$$\therefore {}^{n+1}C_r + {}^{n+1}C_{r-1} = \frac{{}^{n+2}P_r}{r!}$$

61. Answer :

Reason :

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$$\begin{aligned}
 xyz &= \frac{3^{\frac{4}{9}} \times 7^{\frac{2}{7}}}{5^{\frac{2}{3}}} \times \frac{5^{\frac{4}{9}} \times 3^{\frac{1}{3}}}{7^{\frac{3}{7}}} \times \frac{3^{\frac{2}{9}}}{5^{\frac{7}{9}} \times 7^{\frac{6}{7}}} \\
 &= \frac{3^{\left(\frac{4}{9} + \frac{3}{9} + \frac{2}{9}\right)}}{5^{\left(\frac{2}{3} - \frac{4}{9} + \frac{7}{9}\right)} \times 7^{\left(\frac{3}{7} + \frac{6}{7} - \frac{2}{7}\right)}} \\
 &= \frac{3^{\frac{9}{9}}}{5^{\frac{9}{9}} \times 7^{\frac{7}{7}}} = \frac{3}{5 \times 7} = \frac{3}{35}
 \end{aligned}$$

62. Answer :

Reason :

$$\begin{aligned}
 xyz &= \frac{\sqrt[5]{m^3}}{\sqrt{n}} \times \frac{\sqrt[3]{n}}{\sqrt[5]{m^2}} \times \frac{\sqrt[5]{m^4}}{\sqrt[6]{n^5}} \\
 &= \frac{m^{\frac{3}{5}}}{n^{\frac{1}{2}}} \times \frac{n^{\frac{1}{3}}}{m^{\frac{2}{5}}} \times \frac{m^{\frac{4}{5}}}{n^{\frac{5}{6}}} \\
 &= \frac{m^{\left(\frac{3}{5} - \frac{2}{5} + \frac{4}{5}\right)}}{n^{\left(\frac{1}{2} - \frac{1}{3} + \frac{5}{6}\right)}} = \frac{m^{\frac{5}{5}}}{n^{\frac{6}{6}}} = \frac{m}{n}
 \end{aligned}$$

63. Answer :

Reason : Let the numbers be a, b. Let the A.M be A, GM be G and HM be H

As A.M. exceeds GM by 15. So, $A - G = 15$; Or, $(A - 15)^2 = G^2$ (i)

As AM exceeds HM by 27. So, $A - H = 27$ Or, $H = A - 27$

$G^2 = AH = A(A - 27)$ Putting this value of G^2 in equation (i) we get

$$(A - 15)^2 = A(A - 27)$$

$$\text{Or, } -30A + 225 = -27A$$

$$\text{Or, } 3A = 225$$

$$\text{Or, } A = 75. \text{ Or, } (a+b) = 150$$

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So, $G = 60$ Or, $\sqrt{ab} = 60$ i.e $ab = 3600$

So, $a + \frac{3600}{a} = 150$

Or, $a^2 - 150a + 3600 = 0$.

Or, $a^2 - 120a - 30a + 3600 = 0$.

Or, $a(a - 120) - 30(a - 120) = 0$

Or, $(a - 30)(a - 120) = 0$

So the value of the numbers would be (30, 120) or (120, 30)

So the lowest value is 30.

64. Answer :

Reason :

$$z = {}^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7$$

$$\text{Or, } z = \frac{15!}{8!7!} + \frac{15!}{9!6!} - \frac{15!}{6!9!} - \frac{15!}{7!8!}$$

$$= \frac{15!}{8!6!} \left(\frac{1}{7} + \frac{1}{9} \right) - \frac{15!}{8!6!} \left(\frac{1}{9} + \frac{1}{7} \right) = \frac{15! \times 16}{8!6! \times 7 \times 9} - \frac{15! \times 16}{8!6! \times 7 \times 9}$$

$$= \frac{16!}{9!7!} - \frac{16!}{9!7!} = {}^{16}C_9 - {}^{16}C_9$$

Or, $z = 0$

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65. Answer :

Reason : (i) As the desired number of non-official members in the committee would be at least three – there would be three types of groupings – (i) 2 non-official members and 3 official members: This

could be done in ${}^4C_2 \times {}^8C_3 = \frac{3 \times 4}{1 \times 2} \times \frac{6 \times 7 \times 8}{1 \times 2 \times 3} = 56 \times 6 = 336$ ways

(ii) 3 non-official members and 2 official members:

${}^4C_3 \times {}^8C_2 = 4 \times \frac{7 \times 8}{2} = 56 \times 2 = 112$ ways.

(iii) 4 non-official members and 1 official member.

${}^4C_4 \times {}^8C_1 = 1 \times 8 = 8$ ways.

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So, total $(336 + 112 + 8)$ or 456 committees could be formed in the give conditions.

66. Answer :

Reason :

$$\begin{aligned}
 81^{\frac{1}{\log_5 3}} + 27^{\log_9 36} + 3^{\frac{4}{\log_7 9}} &= 3^{\frac{4}{\log_5 3}} + 3^{3\log_9 36} + 3^{\frac{4}{\log_7 9}} \\
 &= 3^{4\log_3 5} + 3^{\frac{3}{2}\log_3 36} + 3^{4\log_3 7} \\
 &= 3^{4\log_3 5} + 3^{\frac{3}{2}\log_3 36} + 3^{\frac{4}{2}\log_3 7} = 3^{\log_3 625} + 3^{\log_3 6^3} + 3^{\log_3 7^2} \\
 &= 3^{\log_3 625} + 3^{\log_3 216} + 3^{\log_3 49} \\
 &= 625 + 216 + 49 = 890
 \end{aligned}$$

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67. Answer :

Reason :

$$\begin{aligned}
 \log_{12} 18 &= a, \text{ and } \log_{24} 54 = b, \\
 ab + 5(a - b) &= \log_{12} 18 \times \log_{24} 54 + 5(\log_{12} 18 - \log_{24} 54) \\
 &= \frac{\log 18}{\log 12} \times \frac{\log 54}{\log 24} + 5 \left(\frac{\log 18}{\log 12} - \frac{\log 54}{\log 24} \right) \\
 &= \frac{\log 18 \times \log 54 + 5 \log 18 \times \log 24 - 5 \log 12 \times \log 54}{\log 12 \times \log 24} \\
 &= \frac{(2 \log 3 + \log 2) \times (3 \log 3 + \log 2) + 5(2 \log 3 + \log 2) \times (3 \log 2 + \log 3) - 5(2 \log 2 + \log 3) \times (3 \log 3 + \log 2)}{(2 \log 2 + \log 3) \times (3 \log 2 + \log 3)} \\
 \text{Let } \log 3 &= x \text{ and } \log 2 = y \text{ then } ab + 5(a - b) = \\
 &= \frac{(2x + y)(3x + y) + 5(2x + y)(x + 3y) - 5(x + 2y)(3x + y)}{(x + 2y)(x + 3y)} \\
 &= \frac{6x^2 + 5xy + y^2 + 10x^2 + 35xy + 15y^2 - 15x^2 - 35xy - 10y^2}{x^2 + 5xy + 6y^2} \\
 &= \frac{x^2 + 5xy + 6y^2}{x^2 + 5xy + 6y^2} = 1
 \end{aligned}$$

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68. Answer :

Reason :

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If $\log_3 2$, $\log_3 (2^x - 5)$ and $\log_3 (2^x - 7/2)$ are in arithmetic progression

$$\text{Then } 2\log_3 (2^x - 5) = \log_3 2 + \log_3 (2^x - 7/2)$$

$$\text{Or, } 2\log_3 (2^x - 5) = \log_3 2(2^x - 7/2) = \log_3 (2^{x+1} - 7)$$

$$\text{Or, } (2^x - 5)^2 = (2^{x+1} - 7)$$

$$\text{Or, } 2^{2x} - 10 \cdot 2^x + 25 = 2^{x+1} - 7$$

$$\text{Let } 2^x = k$$

$$\text{Or, } k^2 - 10k + 25 = 2k - 7$$

$$\text{Or, } k^2 - 12k + 32 = 0$$

$$\text{Or, } k^2 - 8k - 4k + 32 = 0$$

$$\text{Or, } k(k-8) - 4(k-8) = 0$$

$$\text{Or, } (k-4)(k-8) = 0$$

$$\text{Or, } 2^x = 4 \text{ or } 8 = 2^2 \text{ or } 2^3$$

$$\text{So, } x = 2 \text{ or } 3$$

69. Answer :

Reason :

$$\text{Let } \frac{\log a}{(b-c)} = \frac{\log b}{(c-a)} = \frac{\log c}{(a-b)} = k$$

$$\therefore \log a = k(b-c); \log b = k(c-a); \log c = k(a-b)$$

$$\therefore \log a^a b^b c^c = a \log a + b \log b + c \log c = ak(b-c) + bk(c-a) + ck(a-b) = 0$$

$$\text{So, } a^a b^b c^c = 1$$

70. Answer :

$$\text{Reason : } T_m = a + (m - 1) d = n$$

$$\text{And } T_n = a + (n - 1)d = m.$$

$$\text{Solving both equations we get } d = -1 \text{ and } a = m + n - 1$$

$$\begin{aligned} \text{Then } T_{m+n} &= a + (m + n - 1) d \\ &= (m + n - 1) + (m + n - 1) (-1) \\ &= 0. \end{aligned}$$

71. Answer :

Reason :

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$$\frac{S_3}{S_8} = \frac{\frac{a(r^3-1)}{(r-1)}}{\frac{a(r^6-1)}{(r-1)}} = \frac{a(r^3-1)}{a(r^6-1)} = \frac{(r^3-1)}{(r^3+1)(r^3-1)} = \frac{125}{152}$$

$$\text{Or, } \frac{1}{(r^3+1)} = \frac{125}{152}$$

$$\text{Or, } (125r^3 + 125) = 152$$

$$\text{Or, } 125r^3 = 27$$

$$\text{Or, } r^3 = \frac{27}{125}$$

$$\text{Or, } r = \frac{3}{5}$$

72. Answer :

Reason :

$$\text{Let } a^{\frac{1}{bx-c}} = p \quad \text{Or, } \frac{1}{bx-c} = \log_a p = \frac{\log_e p}{\log_e a} = \frac{\ln p}{\ln a}$$

Now differentiating both sides with respect to x

$$\frac{d\left(\frac{1}{bx-c}\right)}{dx} = \frac{1}{\ln a} \times \frac{d \ln p}{dx} = \frac{1}{\ln a} \times \frac{d \ln p}{dp} \times \frac{dp}{dx}$$

$$\text{Or, } -b(bx-c)^{-2} = \frac{1}{\ln a} \times \frac{1}{p} \times \frac{dp}{dx}$$

$$\text{Or, } \frac{dp}{dx} = -b(bx-c)^{-2} \times p \times \ln a$$

$$\text{Now by putting } p = a^{\frac{1}{bx-c}}$$

$$\text{Or, } \frac{d\left(a^{\frac{1}{bx-c}}\right)}{dx} = -\frac{ba^{\frac{1}{bx-c}} \times \ln a}{(bx-c)^2}$$

73. Answer :

$$\text{Reason : } \sqrt{x+y} + \sqrt{y-x} = c$$

Squaring on the both sides, we get

$$X + y + y - x + 2\sqrt{(x+y)(y-x)} = c^2$$

$$2\sqrt{(x+y)(y-x)} = c^2 - 2y$$

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squaring on the both sides,we get

$$4(y^2 - x^2) = c^4 + 4y^2 - 4c^2y$$

$$4c^2y = c^4 + 4x^2$$

$$y = (c^4 + 4x^2)/4c^2$$

$$y = x^2/c^2 + c^2/4$$

$$dy/dx = 2x/c^2 \quad \frac{d^2y}{dx^2} = 2/c^2.$$

74. Answer :

Reason :
$$f(x) = \frac{x^2 - 4x + 3}{x^2 - 1} = \lim_{x \rightarrow 1} \frac{x^2 - 4x + 3}{x^2 - 1} = \lim_{x \rightarrow 1} \frac{(x-1)(x-3)}{(x-1)(x+1)} = \lim_{x \rightarrow 1} \frac{(x-3)}{(x+1)} = \frac{-2}{2} = -1$$

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75. Answer :

Reason :

$$\begin{aligned} f(x) &= \lim_{x \rightarrow 1} \frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3} = \lim_{x \rightarrow 1} \frac{(2x-3)(\sqrt{x}-1)}{2x^2+3x-2x-3} \\ &= \lim_{x \rightarrow 1} \frac{(2x-3)(\sqrt{x}-1)}{(2x+3)(x-1)} = \lim_{x \rightarrow 1} \frac{(2x-3)(\sqrt{x}-1)}{(2x+3)(\sqrt{x}-1)(\sqrt{x}+1)} \\ &= \lim_{x \rightarrow 1} \frac{(2x-3)}{(2x+3)(\sqrt{x}+1)} \end{aligned}$$

Now putting x=1 we get

$$\frac{(2-3)}{5 \times 2} = -\frac{1}{10}$$

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76. Answer :

Reason :
$$\frac{d \ln(a - bx)}{dx} = \frac{d \ln(a - bx)}{d(a - bx)} \times \frac{d(a - bx)}{dx} = \frac{1}{(a-bx)} \times (-b) = \frac{b}{(bx - a)}$$

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77. Answer :

Reason :

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$$\begin{aligned}\frac{d \ln\left(\frac{1}{x+a}\right)}{dx} &= \frac{d \ln\left(\frac{1}{x+a}\right)}{d\left(\frac{1}{x+a}\right)} \times \frac{d\left(\frac{1}{x+a}\right)}{d(x+a)} \times \frac{d(x+a)}{dx} \\&= \frac{1}{\frac{1}{x+a}} \times \{-1 \times (x+a)^{-2}\} \times 1 \\&= -\frac{(x+a)}{(x+a)^2} = -\frac{1}{(x+a)}\end{aligned}$$

78. Answer :

Reason :

$$\frac{d e^{\frac{1}{x+a}}}{dx} = \frac{d e^{\frac{1}{x+a}}}{d\left(\frac{1}{x+a}\right)} \times \frac{d\left(\frac{1}{x+a}\right)}{dx} = e^{\frac{1}{x+a}} \times \{-1(x+a)^{-2}\} = -\frac{e^{\frac{1}{x+a}}}{(x+a)^2} = -\frac{(x+a)\sqrt{e}}{(x+a)^2}$$

79. Answer :

Reason :

$$\frac{d \frac{1}{\sqrt{x-a}}}{dx} = \frac{d \frac{1}{\sqrt{x-a}}}{d(x-a)} \times \frac{d(x-a)}{dx} = -\frac{1}{2}(x-a)^{\left(-\frac{1}{2}-1\right)} \times 1 = -\frac{1}{2(x-a)^{\frac{3}{2}}}$$

80. Answer :

Reason : Profit function, $F = \text{Revenue} - \text{Total cost}$
 $= R - C$

$$= 8Q - \left[1,26,000 + 60\left(\frac{Q}{900}\right)^2 \right]$$

$$F = 8Q - 60\left(\frac{Q}{900}\right)^2 - 1,26,000$$

$$F' = 8 - 60 \times 2 \left(\frac{Q}{900}\right) \times \frac{1}{900}$$

$$\text{or } F' = 8 - \frac{Q}{6,750}$$

$$F' = 0 \Rightarrow 8 - \frac{Q}{6,750} = 0$$

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$$\text{or } \frac{Q}{6,750} = 8$$

$$\text{or } Q = 8 \times 6,750 = 54,000$$

$$F'' = -\frac{1}{6,750}$$

$\therefore F''$ is negative for all values of Q .

At $Q = 54,000$, $F' = 0$ and $F'' < 0$

$\therefore$ The profit is maximum at $Q = 54,000$ units.

81. Answer :

Reason :

$$\lim_{x \rightarrow \infty} \frac{x^2 - 4}{2 + x - 4x^2} = \lim_{x \rightarrow \infty} \frac{(1 - \frac{4}{x^2})}{\frac{2}{x^2} + \frac{1}{x} - 4} = \lim_{x \rightarrow \infty} \frac{(1 - \frac{4}{\infty})}{\frac{2}{\infty} + \frac{1}{\infty} - 4} = -1/4.$$

82. Answer :

Reason :

$$\text{Let } f(x) = \begin{cases} 3^{-x} - 1, & \text{for } -1 \leq x < 0 \\ \frac{x}{x^2 - 2}, & \text{for } 0 \leq x < 3 \\ 2^x - 11, & \text{for } 3 \leq x \leq 6 \end{cases}$$

$$\text{The value of } f(4) = 2^4 - 11 = 16 - 11 = 5$$

83. Answer :

Reason :

$$y = f(x) = \frac{x+2}{x-1};$$

$$f(y) = \frac{\frac{x+2}{x-1} + 2}{\frac{x+2}{x-1} - 1} = \frac{\frac{x+2+2x-2}{x-1}}{\frac{x+2-x+1}{x-1}} = \frac{3x}{3} = x$$

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